Asymptotics of Two-dimensional Singular Perturbation Problems

CHUNHUA OU
Department of Math and Stat, Memorial University, Canada
Email: ou@mun.ca

Since the pioneer work of Norman Levinson [Annals of Mathematics, 1950] on the first boundary value problem for

$$\begin{cases} \epsilon \Delta u + A(x,y)u_x + B(x,y)u_y + C(x,y)u = 0, & (x,y) \in D, \\ u(x,y) = \varphi(x,y), & \text{on } S = \partial D, \end{cases} \tag{1}$$

where ϵ is small, Δ is the Laplace operator and $\Delta u = u_{xx} + u_{yy}$, A, B, C, φ and the boundary ∂D are smooth, there have been a number of extensions of his work including those by Echauss, Jager, Grasman and Nico Temme. These studies were significantly new and interestingly sophisticated since the so-called "ordinary boundary layer", "elliptic boundary layer" or "parabolic boundary layer" were incorporated. However, to simplify the geometry, all of them assumed that in the domain D, there are no any fixed points of the characteristic system

$$\frac{dx}{A(x,y)} = \frac{dy}{B(x,y)}. \tag{2}$$

In this presentation, I will talk about the challenging problem when a fixed point is inside the domain. Based on the geometry, this fixed point could be a stable node, an unstable node, a saddle or even a center. Some special functions including Airy, Hermite and Biconfluent Heun are applied to study the approximation. These results are potentially to be further extended to 3D or Higher dimension case.